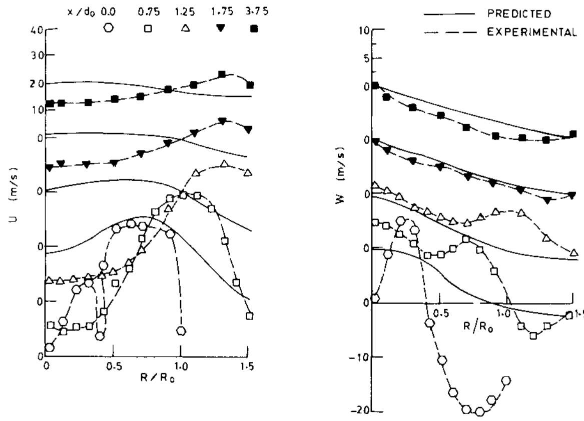

a) Axial velocity U. b) Circumferential velocity W.

Fig. 4 Comparison between experimental and predicted velocity profiles for case A_3.

Predictions for contra- and coswirling flows exhausting in a nonexpanding confinement made by Ramos and Sommer[8] matched only qualitatively. They noted that the introduction of streamline curvature correction might improve their results.

Conclusions

A close comparison of numerical predictions with experimental results for contraswirling jets in a suddenly expanding confinement leads to the following conclusions:

1) Theoretical predictions for the axial component of the velocity have closer agreement with the experimental values for weak swirls. The agreeement becomes poorer with the increase in the intensity of swirl.

2) Theoretical predictions for the circumferential component of velocity when compared with the experimental values show a reverse trend.

3) The LPS correction introduced to account for the streamline curvature improves the results of the prediction technique. The predictions are not satisfactory for high streamline curvatures caused by high swirl intensity and flow expansion.

4) The limitations of the turbulence model is also exposed in the zones having steep velocity gradients and in its inability to pick up the central recirculation zone (case A_3).

References

[1]Singh, S. N., "Some Aerodynamic Investigations of Confined Co-Axial Swirling Jets," Ph.D. Thesis, Indian Institute of Technology, New Delhi, 1985.

[2]Habib, M. A. and Whitelaw, J. H., "Velocity Characteristics of Confined Co-Axial Jets with and without Swirl," *Transactions of ASME, Journal of Fluids Engineering*, Vol. 102, 1980, pp. 47-53.

[3]Khalil, K. H., "Flow Mixing and Heat Transfer in Furnances," *HMT Series*, Pergamon Press, New York, 1978.

[4]Srinivasan, R. and Mongia, H. C., "Numerical Computation of Swirling Recirculating Flow, Final Report," NASA CR-165196, Sept. 1980.

[5]Vu, B. T. and Gouldin, F. C., "Flow Measurements in a Model Swirl Combustor," *AIAA Journal*, Vol. 20, May 1982, pp. 642-651.

[6]Ramos, J. I., "Turbulent Nonreacting Swirling Flows," *AIAA Journal*, Vol. 22, June 1984, pp. 846-847.

[7]Launder, B. S., Priddin, C. H., and Sharma, B. I., "The Calculation of Turbulent Boundary Layers on Spinning and Curved Surfaces," *Transactions of ASME, Journal of Fluids Engineering*, Vol. 99, 1977, pp. 231-239.

[8]Ramos, J. I. and Sommer, H. I., "Swirling Flow in a Research Combustor," *AIAA Journal*, Vol. 23, Feb. 1985.

Relationship Between Pseudocompressible and Unsteady Compressible Flow at Low Mach Numbers

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Nomenclature

a	= sound speed
L	= characteristic length scale
M	= Mach number = u/a
p	= pressure
Re	= Reynolds number = $\rho_\infty L u_R/\mu_R$
t	= time
T	= characteristic time scale
u	= velocity vector
u^*	= reference velocity used to form reference time
β	= "compressibility factor," used in method of pseudocompressibility
γ	= ratio of specific heats
δ	= boundary-layer thickness
θ	= temperature
ρ	= fluid density
τ	= stress tensor

Subscripts and Superscripts

R	= reference quantity
∞	= ambient value
$(\ )'$	= perturbation = $(\ ) - (\ )_\infty$

I. Introduction

THE method of pseudocompressibility[1] has been shown to work very well for the computation of steady viscous incompressible flows (e.g., Refs. 2 and 3). Rogers and Kwak[3] have also used this method for computing vortex shedding by a circular cylinder. They obtained good agreement between predicted streamlines and measurements. The question arises as to how well the method of pseudocompressibility can be expected in general to perform for unsteady-flow analyses, given its utilization of what is ostensibly a fictitious pressure time derivative. The answer to this question lies in the relationship between the differential equations used in this method to those of compressible flow. This relationship is derived below for "external" laminar flows.

II. Background

The differential equations used in the pseudocompressibility method are[3]

$$(1/\beta)p_t + \nabla \cdot \boldsymbol{u} = 0 \tag{1a}$$

$$\boldsymbol{u}_t + \boldsymbol{u} \cdot \nabla \boldsymbol{u} + \nabla p = \nu \nabla^2 \boldsymbol{u} \tag{1b}$$

For flows that have a steady state, $p_t \to 0$ as $t \to \infty$, so that the above equations reduce to those of incompressible flow in the steady state.

In contrast, the equations of compressible flow are[4]

$$\rho_t + \nabla \cdot (\rho \boldsymbol{u}) = 0 \tag{2a}$$

$$\boldsymbol{u}_t + \boldsymbol{u} \cdot \nabla \boldsymbol{u} + \frac{1}{\rho} \nabla p - \frac{1}{\rho} \nabla \cdot \tau = 0 \tag{2b}$$

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$$p_t + u \cdot \nabla p + \gamma p \nabla \cdot u - (\gamma - 1)(\nabla \cdot (k \nabla \theta) + \Phi) = 0 \tag{2c}$$

It will next be shown how the equations of compressible flow compare with those of pseudocompressible flow as the characteristic Mach number goes to zero.

III. The Equations of Compressible Flow at Low Mach Number

To see the role of the characteristic Mach number M_R explicitly, the following dimensionless quantities are formed:

$$x_{\text{ND}} = x_{\text{D}}/L, \quad y_{\text{ND}} = y_{\text{D}}/\delta, \quad z_{\text{ND}} = z_{\text{D}}/L, \quad t_{\text{ND}} = t_{\text{D}}/T$$

$$\rho_{\text{ND}} = \rho'_{\text{D}}/\rho'_{\text{R}} = (\rho_{\text{D}} - \rho_\infty)/\rho'_{\text{R}}$$

$$p_{\text{ND}} = p'_{\text{D}}/p'_{\text{R}} = (p_{\text{D}} - p_\infty)/p'_{\text{R}}$$

$$\theta_{\text{ND}} = \theta'_{\text{D}}/\theta'_{\text{R}} = (\theta_{\text{D}} - \theta_\infty)/\theta'_{\text{R}}, \quad u_{\text{ND}} = u_{\text{D}}/u'_{\text{R}}$$

$$M_{\text{R}} = u_{\text{R}}/a_{\text{R}}, \quad \Phi_{\text{ND}} = (\gamma - 1)\Phi_{\text{D}}/(\mu_{\text{R}} u_{\text{R}}^2/\delta^2) \tag{3}$$

The primed variables represent perturbations of dimensional quantities from ambient. The ND and D subscripts denote dimensionless and dimensional quantities, respectively. Though the velocities are scaled by u_{R}, the characteristic (reference) velocity (e.g., u_∞ in external flows) u^*, the velocity related to the characteristic time scale T such that $L/T = u^*$, may be chosen differently. If $u^* = a_{\text{R}}$, the reference sound speed, then T is of the order of the time required for an acoustic wave to travel the characteristic distance L. If $u^* = u_{\text{R}}$, then T is of the order of the time required for a fluid particle to travel L. (Other time scales are of course also possible; e.g., one could have chosen the viscous time scale δ^2/ν, which would result from setting $u^* = \nu/\delta$.) Depending on the choice of u^*, the dimensionless time-derivative terms (e.g., u_t) may or may not be 0(1). All other dimensionless variables are 0(1), by the proper choices for the reference quantities u_{R}, p'_{R}, etc. The temperature θ_{D} is related to the pressure and density via the perfect gas law, $p_{\text{D}} = \rho_{\text{D}} R \theta_{\text{D}}$ (R is the gas constant). At high Reynolds numbers ($Re = \rho_\infty L u_{\text{R}}/\mu_{\text{R}}$), i.e., in boundary layers, L/δ is $0(Re^{1/2})$ and reduces to 0(1) as the Reynolds number is lowered. We also bear in mind that at low Mach numbers the order of the density perturbations is given by $\rho'_{\text{D}}/\rho_\infty = 0(M_{\text{R}}^2)$ and similarly for the pressure and temperature perturbations, p'_{D} and e'_{D}, respectively. We therefore choose the following reference perturbation quantities: $\rho'_{\text{R}} = M_{\text{R}}^2 \rho_\infty$, $p'_{\text{R}} = p_\infty$, $u_{\text{R}}^2 = \gamma M_{\text{R}}^2 p_\infty$, and $\theta'_{\text{R}} = (\gamma - 1) M_{\text{R}}^2 \theta_\infty$. This choice of scaling is based on the assumption that, in the region of interest, the density, pressure, and temperature are composed of their freestream values plus acoustic perturbations. This assumption should be valid for external flow problems. Making use of this information, along with $L/\delta = Re^{1/2}$, on substitution of Eq. (3) into Eq. (2) and dropping the ND subscripts, we get, to second order in M_{R},

$$\rho_t + \left\{\frac{u_{\text{R}}}{u^*}\right\}\left\{\left(\frac{1}{M_{\text{R}}^2}\right)\nabla \cdot u + u \cdot \nabla \rho\right\} \approx 0 \tag{4a}$$

$$u_t + \left\{\frac{u_{\text{R}}}{u^*}\right\}\{u \cdot \nabla u + \nabla p\} \approx \left\{\frac{u_{\text{R}}}{u^*}\right\}\{Re^{-1}(u_{xx} + u_{zz}) + u_{yy}\} \tag{4b}$$

$$p_t + \left\{\frac{u_{\text{R}}}{u^*}\right\}\left\{\frac{1}{M_{\text{R}}^2}\right\}\nabla \cdot u \approx \left\{\frac{u_{\text{R}}}{u^*}\right\} \times \left\{-u \cdot \nabla p + \Phi + \frac{(\gamma - 1)}{Pr}[\theta_{yy} + Re^{-1}(\theta_{xx} + \theta_{zz})]\right\} \tag{4c}$$

If we choose $u^* = a_{\text{R}}$, Eq. (4) becomes

$$\rho_t + M_{\text{R}}\left\{\left(\frac{1}{M_{\text{R}}}\right)^2 \nabla \cdot u + u \cdot \nabla \rho\right\} = 0 \tag{5a}$$

$$u_t + M_{\text{R}}\{u \cdot \nabla u + \nabla p\} = M_{\text{R}}\{Re^{-1}(u_{xx} + u_{zz}) + u_{yy}\} \tag{5b}$$

$$p_t + \left(\frac{1}{M_{\text{R}}}\right)\nabla \cdot u = M_{\text{R}}\left\{-u \cdot \nabla p + \Phi + \left(\frac{\gamma - 1}{Pr}\right)[\theta_{yy} + Re^{-1}(\theta_{xx} + \theta_{zz})]\right\} \tag{5c}$$

while if $u^* = u_{\text{R}}$, we obtain

$$\rho_t + \left(\frac{1}{M_{\text{R}}}\right)^2 \nabla \cdot u + u \cdot \nabla \rho = 0 \tag{6a}$$

$$u_t + u \cdot \nabla u + \nabla p = \{Re^{-1}(u_{xx} + u_{zz}) + u_{yy}\} \tag{6b}$$

$$p_t + \left(\frac{1}{M_{\text{R}}}\right)^2 \nabla \cdot u = -u \cdot \nabla p + \Phi + \left(\frac{\gamma - 1}{Pr}\right)[\theta_{yy} + Re^{-1}(\theta_{xx} + \theta_{zz})] \tag{6c}$$

IV. Comparison of the Equations of Low Mach Number and Pseudocompressible Flow

We are now in a position to compare the equations of compressible flow at low Mach numbers to those of pseudocompressible flow. Let us first consider the inviscid case. For example, Eq. (6c) becomes

$$p_t + (1/M_{\text{R}})^2 \nabla \cdot u + u \cdot \nabla p = 0 \tag{7}$$

It is seen that Eqs. (6b) and (7) are independent of ρ and involve only u and p; i.e., for time-accurate computations of unsteady, inviscid, low-Mach-number flow, the continuity equation uncouples from the other two equations (momentum and energy), so that one can solve for the velocity and pressure independently of the density fluctuations. Furthermore, comparison of Eqs. (6b) and (7) with Eqs. (1), we see that the former reduces to the latter if $\beta = 1/M_{\text{R}}^2$ and if, in the energy equation (7), we neglect the term $u \cdot \nabla p$. It is seen that the method of pseudocompressibility tries, in effect, to solve the unsteady energy equation, not the unsteady continuity equation. However, $u \cdot \nabla p$ is, in general, not negligible. Since there is no reason to expect the velocity and the pressure gradient always to be orthogonal (if they were, in steady flows the pressure would remain constant along streamlines), $u \cdot \nabla p$ does not vanish identically. Furthermore, because of our scaling, both u and ∇p are 0(1), so that $u \cdot \nabla p = 0(1)$, in general.

Let us next examine the terms of the continuity equation (6a). We note that in the steady state, we have

$$u \cdot \nabla \rho = -(1/M_{\text{R}})^2 \nabla \cdot u \tag{8}$$

Now, for inviscid low-Mach-number flow

$$p=\rho+0(M_R^2) \tag{9a}$$

Therefore, to $0(M_R^2)$

$$u\cdot\nabla\rho=u\cdot\nabla p \tag{9b}$$

$$\rho_t=p_t \tag{9c}$$

and

$$u\cdot\nabla p=-(1/M_R)^2\nabla\cdot u \tag{9d}$$

From Eq. (9d) we conclude that $\nabla\cdot u=0(M_R^2)$. If we combine Eqs. (9b), (9c), and (6a), we get Eq. 7. This means, that to $0(M_R^2)$, the energy and continuity equations are identical in inviscid regions of the flow.

The above analysis partially explains why conventional compressible flow computations break down at low Mach numbers—in the steady state, the continuity equation and the inviscid part of the energy equations become very similar, leading to ill conditioning of the numerics (also, the use of p and ρ instead of p' and ρ' and the use of total rather than perturbation scaling quantities in conventional compressible computations produce a high noise-to-signal ratio).

Let us next consider the viscous case. Only if there is no heat addition, the walls are cold and k is small, can the terms in the energy equation involving the temperature be safely neglected. Doing so would maintain the uncoupling of the continuity from the other two equations.

V. Conclusions

The method of pseudocompressibility can be regarded as one of computing incompressible flows by replacing the incompressible continuity equation with part of the low-Mach-number energy equation. For the time-accurate calculation of unsteady flows, the terms absent from the pseudocompressible continuity/energy equation are, in general, not negligible; numerical experiments should be carried out to compare the differences between using Eqs. (1) and (5) or Eq. (6).

To use Eqs. (5b) and (5c), or (6b) and (6c), one should choose a value of M_R low enough so that the flow is in effect incompressible (truly incompressible flows do not exist in nature anyway; it is assumed that when the Mach-number-like parameter of any Navier-Stokes flow of a perfect gas, a liquid, or whatever, is sufficiently low, then this flow is effectively similar to a truly incompressible one), but not so low that the term $1/M_R^2$ in Eqs. 5 is troublesome or that the approximate-factorization error[2,3] is too large. If the numerical algorithm used to solve Eqs. (5) does not make use of approximate factorization or, if it corrects for it, then the upper bound for $1/M_R^2$ discussed in Refs. 2 and 3 can be relaxed and lower Mach numbers can be safely utilized.

References

[1]Chorin, A. J., "A Numerical Method for Solving Incompressible Viscous Flow Problems," *Journal of Computational Physics*, Vol. 2, 1967, pp. 12-26.

[2]Chang, J. L. C. and Kwak, D., "On the Method of Pseudo Compressibility for Numerically Solving Incompressible Flows," AIAA Paper 84-0252, Jan. 1984.

[3]Rogers, S. E., Kwak, D., and Kaul, U., "On the Accuracy of the Pseudocompressibility Method in Solving the Incompressible Navier-Stokes Equations," AIAA Paper 85-1689, July 1985.

[4]White, F. M., *Viscous Fluid Flow*, McGraw-Hill, New York, 1974, Chap. 2.

Turbulent Mixing in Nonsteady Jets

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Nomenclature

d	= nozzle diameter
K_1, K_2	= constants
$L(t)$	= instantaneous flame length
Re	= Reynolds number
t	= time
U_{cl}	= jet centerline speed
Ve	= jet entrainment speed
$U_j(t)$	= nozzle exit speed
x	= downstream distance from nozzle exit
$\Gamma(t)$	= circulation
$\delta(t)$	= instantaneous maximum visible flame width
ϕ	= equivalence ratio
Ω	= vorticity

Introduction

THE mixing behavior of the steady jet has been extensively studied. The mean concentration of any conserved scalar ejected from the jet nozzle declines as x^{-1} as the jet mixes with the ambient fluid, where x is the downstream distance. Therefore, one expects that any molecular scale mixing occurring as the fluid proceeds downstream will initially produce mixtures rich in injected fluid near the nozzle and progressively leaner mixtures further downstream. Such behavior has been observed in laboratory experiments.[1-3] It is clear from this that the steady jet mixes the two fluids together over a wide range of mixture ratios. If some mixture ratios are undesirable, the steady jet is assured of producing at least some undesirable mixing. Thus, if the mixing rate could be modified by some means to preferentially mix fluid at a single mixture ratio, rather than the broad range covered by the steady jet, then the amount of mixing at the undesirable mixture ratios could be reduced.

We propose that a nonsteady jet may modify the molecular mixing rate by altering the ratio of ambient to nozzle fluid ingested into turbulent vortices. The notion is fairly simple. Consider the starting jet illustrated in Fig. 1. If the nozzle exit velocity $U_j(t)$ is a step function in time, a toroidal starting vortex is produced. As it convects away from the nozzle, it grows, partially by entrainment of ambient fluid and partially by incorporation of overtaking nozzle fluid from the rear. While, to our knowledge, the ratio of the two contributing components has not been measured for the jet, the related quantity for the starting buoyant plume is reported by Turner[4] to be approximately unity. That is, about equal rates of ambient and nozzle fluid are incorporated into the starting vortex for the plume.

In the jet, the incorporation ratio of ambient to nozzle fluid may be changed by nonsteady effects. At any instant, the vortex has circulation $\Gamma(t)$ and characteristic dimension $\delta(t)$. Thus, it has a finite-volume entrainment appetite of order $\Gamma\delta(t)$. If the flow out of the nozzle is continually accelerated so that the vortex engulfment[5] appetite is completely satisfied by this stream, then negligible ambient fluid would be en-

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